Novel Approach to Quantization of Hodgkin-Huxley Theory of Action Potential: Revival and Renewal of Mathematical Philosophy

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ABSTRACT— A careful analysis of experimental studies carried by Hodgkin- Huxley and Hodgkin-Katz for finding out the electrical impulse produced by sodium ions in the squid axon was done. They described their findings quantitatively in form of a mathematical model for the generation of potential necessary for producing action to be responsible for nerve excitation and current conduction. We are presenting innovative Quantized Model of their theory of impulse generation. It will provide new research platforms in exploring the theory from the point of view of Quantization principles and will make the use of electrodynamical and quantum field theoretical approaches possible in fresh analysis of Hodgkin-Huxley equations and their applications. Some plausible applications and some new conjectures regarding the use of Quantized model will be discussed. This can be thought as an effort supplemented to Revival and Renewal of Mathematical Philosophy.

Key Words — Quantization, Special mechanism, Current conduction in neurons, Hodgkin-Huxley equations, Message transfer in neurons, Quantized model.

1 INTRODUCTION

We present a new way of quantization of the important set of theoretical equations suggested by the team of Hodgkin-Huxley where they have calculated the total applied current I as the sum of four currents contributed by sodium,potassium ions and another ion representing leak current in their studies and developed the set of equations.

We are targeting an important observation of the team of Hodgkin-Huxley that potential in membrane is effected by impulses generated by message carried by ions of sodium gets preference in passing through the excited membrane faster as compared to ions of potassium or ions of chlorine and presenting it in a new perspective which we found through literature search that no other researchers have thought ever on these lines.

Our approach to quantize Hodgkin-Huxley equations for theory of action potential may be thought as Revival and Renewal of Action Potential Theory from Quantization point of view and consequently will be termed as Revival and Renewal of Mathematical Philosophy as applied to exploring new approaches to some existing problems.

We will also highlight the probable and suggested solutions to some of the other problems which were observed by research into the neuronal activities in the brain and nervous system.

In this study effort has been made to quantize Hodgkin-Huxley theory of action potential by quantizing the central equation of Hodgkin-Huxley theory and the associated equations as well as the equations giving the relationships of constants. This innovation shall definitely open new research platforms in exploring the theory for new understanding. Moreover this quantization will make the use of electrodynamical and quantum field theoretical approaches possible in analysis of Hodgkin-Huxley equation and their applications.

The novel idea was conceived when we thought about various quite fundamental questions such as what is the nature of Life, Mind, Soul, Brain and Brain functions, Consciousness, Memory, Life after death etc and some other remarkable brain functions. Searching for understanding of these fundamental paradoxical unfathomed mysteries led towards a new look on the work of Hodgkin-Huxley and Hodgkin-Katz teams.

In the research carried out by Hodgkin-Huxley on conduction of neurons revealed that during the process of conduction of Na^+, K^+ , and other ions the conduction of sodium ions takes precedence over conduction of other ions [1]. Later on Hodgkin-Huxley and Hodgkin-Katz teams also observed the phenomenon of Na^+ ion having more potential passing through the ionic channel before K^+ , and other ions [1], [2], [3]. These observations encouraged and subsequently forced the researcher to think and express the observed phenomenon in terms of presented innovation and consequently express them in terms of quantum theoretical approach to identify the causes. For the purpose efforts are made to develop a quantized model that probably will address the issues effectively and also open rooms for the upcoming researchers. In doing so we came up with new explanations and fresh conjectures based on our innovative approach for some of these not fully understood mysteries regarding the nature of Life, Mind, Soul, Brain and Brain

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functions, Consciousness, Memory, Life after death which are embodied in this paper.

2 LITERATURE REVIEW

A minute and careful study and analysis of the material in three papers [1], [2], [3] revealed that there is a possibility of presenting a novel quantized Hodgkin-Huxley equations model. Hodgkin- Huxley had studied "the effect of sodium ions on the electrical activity of the giant axon of the squid " [1] " The components of membrane conductance in the giant axon of Loligo"[2] and "A quantitative description of membrane current and its application to conduction and excitation in nerve" [3] and from these studies they have developed action potential theory in giant squid. [1], [2], [3].

In their study they came to observe that there are a number of parameters affecting the flow of current carried by sodium, potassium and other ions. Further they have mentioned at several places that potential in membrane is effected by impulses generated by message carried by ions of sodium gets preference in passing through the excited membrane faster as compared to ions of potassium or ions of chlorine. They also clearly identified that there is some effect in this traversing of charges carried by the ions and molecules.[3]

In this paper we present a new approach to understand and explain why this is so by suggesting a new possibility of this happening though they themselves only referred to this strange behavior of some ions having a sort of preference over the other ions by suggesting that there could be a "special mechanism which allows sodium ions to traverse the active membrane at a much higher rate than either potassium or chloride ions". [1]

We also noticed some very strange and puzzling sort of findings that in their study and their deliberations on it they have mentioned interspersed and replete throughout these papers [1], [2], [3] such phenomenon as we find in Quantum Theory, Quantum Field Theory, and Particle Physics. Some of these interspersed phenomenon were depicted by the use of very peculiar terminology of the field of Quantum Theory, such as carrying of messages by impulses (waves) which cannot be measured accurately, difficulties faced in recording changes in the behavior of nerves to carry messages, dependences on the concentrations of different kinds of ions (charged particles) and the presence of electric fields and others such phenomenon to mention just a few of them. Such observations which were found throughout in the said papers [1], [2], [3] prompted us to look at the theory of Action Potential developed in the form of a set of equations widely known as Hodgkin–Huxley Equations from a new perspective of a Quantization and consequently we are presenting in this work our suggested Quantized Model of Hodgkin –Huxley Equations.

3 METHODOLOGY

Our approach to the development of the Quantized model for Hodgkin-Huxley Equations will be theoretical formulization using the principles of mathematics and physics and a brief but succinct background of Quantization Theory with a discussion of application possibilities of the newly developed Quantized model.

This is fundamentally a theoretical approach based on new crisp logical and philosophical thinking in the field of Theory of Action Potential as developed by Hodgkin-Huxley. The results, discussion and conclusions all will be based on plausible point of view based on process of logical, philosophical and mathematical thinking.

3.1 Brief Background of Quantization Theory

As we know that Quantization is the concept that a physical quantity can have only certain discrete values. Electrical charge, energy, light, angular momentum, and matter are all quantized on the microscopic level.

For example, matter is quantized because it is composed of individual particles that cannot be subdivided; it is not possible to have half an electron, half sodium atom, half potassium atom, etc. Also, the energy levels of electrons in atoms are quantized. The electron's energy can have only certain values, and all intermediate values are prohibited.

We will use the basic theory of current, potential, conductance, charge, and energy and give a new formulation of central equation of Hodgkin-Huxley theory of action potential. Our approach will open the newer methods of analysis of Hodgkin-Huxley equations from the perspective of electrodynamical, and quantum field theoretical approaches.

Quantization has a long history and took a proper footing starting from the simple and innocuous but revolutionary observations of Max Planck about the radiation from those bodies which are black, the property of light shining on metals causing electrons emitted from these metals later came to be known as photoelectric effect. Initial investigations, experiments and monumental efforts of a long line of extraordinary physicists including Heinrich Hertz, Robert Millikan, J. J. Thomson, Philipp Lenard and others paved the way culminating in for the phenomenon and theory of photoelectric effect properly theorized by Albert Einstein and winning him Nobel Prize. This was the beginning of a revolution subsequently called Quantum Era.

History of Quantum Theory is based on a kind of strange interactions among mathematicians, theoretical physicists, experimental physicist teams that there have been various bouts of solid predictive activities on part of mathematicians and theoretical physicists and with time lapse confirmations of these qualitative predictions by experimental physicists and the other way round that experimental physicists through their experiments reported their experimental findings and mathematicians and

theoretical physicists found their explanations through fitting well-designed mathematical formulations.

Quantum Mechanics with its various interpretations including Copenhagen , Wave mechanical, Matrix mechanical, Amplitude, Phase Space, Modal, Many Worlds, Path Integral, etc resulted in the consequent development of Quantum Electrodynamics (QED), Quantum Chromodynamics (QCD), and Quantum Field Theory(QFT). Quantization principles development has had quite a checkered history. Some of the major quite prominent players but not all of them in this development and in this order have been Max Born, Niels Bohr, Max Planck, de Broglie, Dirac, Albert Einstein , Heisenberg, Schrodinger, Pauli, Feynman, Fermi, Anderson. Zeeman, Ehrenfest.

Earlier masters have been Maxwell, Faraday, Kirchhoff, Boltzmann Thomas Young, Henri Becquerel, Rutherford, Stern, Gerlach, Josef Stefan, Satvendra Bose. Theoretical, mathematical, philosophical and logical foundations of Quantum theory were contributed by a number of mathematicians and theoretical physicists including David Hilbert, Hamilton, Lagrange, Balmer, Rydberg, von Neumann, Lorentz.

It all started approximately in 1830 with the fundamental enquiries done by Michael Farady into the nature of electromagnetism, which was given theoretical and mathematical rigor in 1865 by the giant James Clerk Maxwell and showed that magnetic and electric fields travel in space not in particles form but in waves form. Gustav Kirchhoff and later on Max Plank's work on radiation from black surfaces paved the way for further necessity of more researches into propagation of electric and magnetic fields in space. Later on geniuses of Ludwig Boltzmann and Max Planck suggested the possibility of discreteness of energy propagation in physical world.

Further down the line of research took a new turn by the research of Heinrich Hertz and later on its theorizing by Albert Einstein of phenomenon of photoelectric effect which neatly combined the previous works of Faraday, Maxwell, Boltzmann and Planck into one grand theory and winning of Nobel Prize for Einstein.

According to quantum theory electrons are bounded to the atom by the electroweak force but each electron remains in motion in terms of spin and angular momentum and carries quantized energy.

The electron does not move freely. It is held firmly in its orbit by the force of attraction exerted by the nucleus called the weak force of attraction in quantum theory consequently according to the principles of quantum phenomena energy carried by the electron is quantized and is given as the following in case of its particle nature:

$$E_s = -2m(e^2)^2\left(\frac{\pi N}{sh}\right)^2, \quad s = 1, 2, 3, 4, \ldots (1)$$

Where e is the charge carried by the electron, s is quantum number, N is the atomic number, h is the Planck's constant and m is the electron mass. And energy carried by the electron in case of its wave nature is given in terms of frequency of the electronic wave and is quantized as:$E = hf$, where f is the frequency and h is Planck's constant.

4 ORIGINAL MODEL OF HODGKIN-HUXLEY EQUATIONS

Hodgkin-Huxley summarized their theory [3] p518,519 in the form of the following set of four equations and associated constants h, n and m which are dimensionless and coefficients α and β which are time-dependent upon potential of membrane depicted in equations labeled (1) – (10):

$$I = C_M\frac{dV}{dt} + \bar{g}_K n^4(V - V_K) + \bar{g}_{Na} m^3 h(V - V_{Na}) + \bar{g}_l(V - V_l) \quad (1)$$

$$\frac{dn}{dt} = \alpha_n(1 - n) - \beta_n n \quad (2)$$

$$\frac{dm}{dt} = \alpha_m(1 - m) - \beta_m m \quad (3)$$

$$\frac{dh}{dt} = \alpha_h(1 - h) - \beta_h h \quad (4) \quad [3] \text{ P } 518$$

$$\alpha_n = \frac{0.01(V + 10)}{\exp\left(\frac{V+10}{10}\right) - 1} \quad (5)$$

$$\alpha_m = \frac{0.1(V + 25)}{\exp\left(\frac{V+25}{10}\right) - 1} \quad (6)$$

$$\alpha_h = 0.07\exp\left(\frac{V}{20}\right) \quad (7)$$

$$\beta_n = 0.125\exp\left(\frac{V}{80}\right) \quad (8)$$

$$\beta_m = 4\exp\left(\frac{V}{18}\right) \quad (9)$$

$$\beta_h = \frac{1}{\exp\left(\frac{V+30}{10}\right) + 1} \quad (10) \quad [3] \text{ P } 519$$

5 QUANTIZED MODEL PRELIMINARIES

Team of Hodgkin-Huxley working in early 1950-60 through their experiments on current flow in squid axon developed a model to show how electrical potential is generated and flows inside axon of squid. Their thrust was on developing differential equations to understand this process of electrodynamics of current flow.

In this paper we are proposing a new model based on quantization principles for understanding the processes being going on inside the neurons for carrying messages. In this new approach we are suggesting a new set of differential equations which involve amount of charges and energies carried by the sodium, potassium, and the ion representing the leak current and therefore we are proposing the new set of quantized equations which follow.

The understanding and prediction of the properties of matter at the atomic, molecular level shows that these follow properties according to the theory of quantum principles. So in our opinion the theory developed to describe the behavior of electrons, atoms and molecules in the fields of quantum mechanics, particle physics and

quantum physics should be applicable to the set of Hodgkin-Huxley equations.

6 QUANTIZED MODEL OF HODGKIN-HUXLEY EQUATIONS

Using the basic principles of Physics , we take: $V - V_K = E_K/Q_K$, $V - V_{Na} = E_{Na}/Q_{Na}$, $V - V_l = E_l/Q_l$ and $V = \frac{E_c}{Q_c}$as the net quantized contribution by potassium, sodium, leak ions, and membrane current respectively and we write equation (1) of original model of Hodgkin-Huxley above as:

$I = I_C + \bar{g}_K n^4 E_K/Q_K + \bar{g}_{Na} m^3 h E_{Na}/Q_{Na} + \bar{g}_l E_l/Q_l$(i)

and consequently equation (1) is changed into an equation of quantized energy

where E_{Na}, E_K, and E_l are the quanta of energy carried by sodium ion , potassium ion , and any other ion e.g. chlorine, etc. respectively and Q_{Na}, Q_K, and Q_l are the quanta of charges carried by sodium ion , potassium ion , and any other ion e.g. chlorine, etc. respectively.

Representing $I = H$, $\bar{g}_K = \bar{s}_K$,$\bar{g}_{Na} = \bar{s}_{Na}$, $\bar{g}_l = \bar{s}_l$, n = b, $m = a$, $h = f$, and

$E_K/Q_K = T_{qK}$, $E_{Na}/Q_{Na} = T_{qNa}$, $E_l/Q_l = T_{ql}$, $\frac{E_c}{Q_c} = T_{qM}$ as the net quantized contribution from potassium, sodium, and leak ions and stable quantized contribution of resting membrane:

We write (i) in more compact form as:

$H_{qF}(t) = H_C + \bar{s}_K b^4 T_{qK} + \bar{s}_{Na} a^3 f T_{qNa} + \bar{s}_l T_{ql}$

where H_{qF} is new quantized functional current, and $T_{qK} = E_K/Q_K$, $T_{qNa} = E_{Na}/Q_{Na}$, $T_{ql} = E_l/Q_l$ are the total quantized contributions of quantum energy divided by the contributions of quantized charge , $\bar{s}_K$, $\bar{s}_{Na}$, and $\bar{s}_l$ represent the constant conductances of potassium, sodium and leakage , b, and a represent potassium and sodium activation variables, , f represents the inactivation variable for sodium. And the remaing three differential equations expressing the relationship for undimensional constants h, n and m similarly will be written as:

$\frac{dn}{dt} = \alpha_b((T_{qK})_a)(1 - b) - \beta_b((T_{qK})_a)b$ (ii)

$\frac{dm}{dt} = \alpha_a((T_{qNa})_a)(1 - a) - \beta_a((T_{qNa})_a)a$ (iii)

$\frac{dh}{dt} = \alpha_f((T_{qNa})_a)(1 - f) - \beta_f(T_{qNa})_a)f$ (iv)

Finally the time-dependent coefficients α and β will take the following compact quantized representation:

$\alpha_n = \frac{0.01(T_{qM} + 10)}{\exp\left(\frac{T_{qM}+10}{10}\right) - 1}$ (v)

$\alpha_m = \frac{0.1(T_{qM} + 25)}{\exp\left(\frac{T_{qM}+25}{10}\right) - 1}$ (vi)

$\alpha_h = 0.07 \exp\left(\frac{T_{qM}}{20}\right)$ (vii)

$\beta_n = 0.125 \exp\left(\frac{T_{qM}}{80}\right)$ (viii)

$\beta_m = 4 \exp\left(\frac{T_{qM}}{18}\right)$ (ix)

$\beta_h = \frac{1}{\exp\left(\frac{T_{qM}+30}{10}\right) + 1}$ (x)

The equations (i) - (x) describe our proposed novel Quantized Model of Hodgkin-Huxley equations. The rest of the solutions will be based upon the calculations carried on by taking into account of new quantized contributions of charge carried by respective ions. We will now think in terms of amount of ionic charge found in solutions used in the experiments which can be performed using the traditional method of stoichiometric calculations. This new formulation now can be used to explain some of the previously unexplaied observations.

7 RESULT AND DISCUSSION

A possible new explanation can now be provided why salt in food among other factors is responsible for development of high blood pressure. As the Quantized Model shows that amount of current is inversely proportional to the number of atoms present so the delivery of sodium ion is faster than the delivery of phosphorous and consequently its sodium effect develops quicker than phosphorous effect because sodium has less numbers of electrons as compared to phosphorous.

The work of Hodgkin-Huxley opened new channels for designing measurable models of studying the mode of signal generation by different types of neurons. In fact the original model studied and developed by Hodgkin-Huxley was dependent upon difference in voltage at one point but to apply the said model in studying the transfer of message one must be able to find the manner of propagation of current through special neuronal processes axons and dendrites.

This manner of propagation can be possibly better understood in the form of Quantized Model of carrying current. The work of Wilfred Rall which was based on the classical theory intended for cables carrying telegraphic messages was also instrumental in understanding current conduction in neuronal process dendrite. In this case also method of quantization of carrying current can result in improved understanding.

If drug therapy is to be developed for various diseases then one of the important ingredients is transferring information to the neurons and other cells so that these neurons and cells follow the message and transfer it faithfully to the designated other neurons and cells, and in our approach it would be possible as information at the level of neurons and cells has been quantized and therefore it will process the information in quantum information processing principles.

In an article [14] it was pointed out that it is very puzzling as to the reasons behind the noise found in the background of neural messaging in the brain and its spontaneity in its production and its usefulness in the new concept of processing of information carried by neurons. Both of these can be understood in the light of this new quantization as presented in this paper.

As Hodgkin-Huxley work stimulated the idea of working pattern of neurons in animals (squid) which paved

the way for further understanding of even more complicated activities of some other neuronal channels. In that case also this new quantization will help in deciphering of actual workings of these other new channels.

Newer techniques of providing cure in technologies connecting machines and brains will be made available. Prospects of replacement of neurons and nerve fibers will be bright if we succeed in developing process of deciphering signals produced in brain and capturing the impulses generated in neurons using the knowledge and principles of genetics and optics.

We hope that model developed here will also be in some way will become useful for medical practitioners in further development of computational aspects of neurology. This quantization will definitely be a added advantage in the progress of discipline of clinical neurological studies. New quantized approach suggested in our work will help deciphering of these questions.

This innovative philosophical and logical thinking on quantization principle will also be beneficial in the improvements of drugs deliveries in the body affecting neurons of every type including the cholinergic neuron, the adrenergic neuron and affecting various physiological systems including autonomic nervous system, central nervous system and other systems.

The process of message transfer between cells representing nerves , muscles and glands receiving the impulses happens when relevant signals produced by chemicals often designated as neuronal transmitters originate in the terminals of the nerve. In this way the delivery be re-evaluated in terms of new thinking of quantum burst of energies which will carry with it the necessary amounts of drugs. This novel approach possibly will open new patterns of thinking for researchers in this field. [14]

8 CONCLUSION

The special mechanism mentioned in Literature Review now can be explained using our novel approach discussed in detail above. The atomic number of the sodium Na^+ is 11 whereas atomic number of potassium K^+ is 19 so the quantity of charge carried by sodium is less as compared to to the quantity of charge carried by potassium atom and as we have quantized the central equation of Hodgkin-Huxley equation where quantity of charges are inversely proportional so this theoretically provides explanation for the observation made by Hodgkin-Huxley that ions of sodium gets preference in passing through the excited membrane faster as compared to ions of potassium or ions of chlorine.

Since the model has been quantized wave and particle nature of ions can be exhibited inside the body and may be responsible for messages transfer in quantized form. Diseases can be given explanations of disturbances in quantum behavior of ions and molecules.

8.1 Some New Conjectures

We humans are probably wonderful specimens of classical physical and quantum physical systems. In some of our activities we are specimens of classical physical systems that is Newtonian physics for example holding of objects by hand. On the other hand our brain and nervous system follow quantum physical system. In short we are a mixture of both classical world and quantum world.

Our new Quantized Model may even be possible to provide a sort of explanation in the working of brains thought processes which may also be a contributing factor in producing on one hand giants like Einstein, Gauss, Archemedies, Newton etc. and on the other hand ordinary persons like Tom, Dick,Harry.This can be termed as an unending duality of our nature incorporating both macro and micro aspects of our body system.

Since in our quantized form charges carried by ions, atoms and molecules and energies quanta present work in form of quantum principles, therefore this new procedure of quantization behavior of molecules, atoms and ions can be utilized for creating new models of messages and information conduction through neural computing techniques and thereby developing new quantum computing of the brain processes and understanding of such mysteries as the nature of dreams, the secrets of consciousness and What is death?

We think probably we can also develop new methods and processes for curing diseases and ailments of nervous system and brain including such diseases as epilepsy, Alzheimer and others by developing a new quantized system of delivery of curing drugs to specific positions.

It takes the nature of waveform which creates flashes in patients. If it corrects itself and propagates in form of ions, that is in particle form then it will correct itself and possibly can cure the patient and patient recovers by himself or by herself as the case may be.

We are invoking all the knowledge accumulated from quantum mechanics, QED, QFT, QCD, and electromagnetic forces. We have only presented the very basic fundamental principles in this formulation and will continue for further probing of ideas and invite other researchers to look into this new innovation in theory of Action Potential and its wide ranging implications in drug therapy, information technology, and quantum computing.

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